Play and Mathematics

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Abstract:
In this paper, I argue that a lack of play and joy in classrooms could be due to our North American standardized education system, which emphasizes achievement outcomes. I argue that this system does not benefit the majority of students, nor the field of mathematics. Many students are negatively affected—both emotionally and academically—by a focus on results. Rather than outcome-driven pedagogy, a focus on learning to enjoy doing mathematics might change the conversation. A kinder, process-driven approach through mathematical play may spark enjoyable teaching and learning. Play (Gadamer, 1960/1989; Huizinga, 1944/1949) has the potential to absorb learners as they seek answers to fun yet challenging mathematics problems. The experience of flow is similar to that of play; when playing, learners get a chance to practice and elaborate on their existing skills in manners that suspend notions of time. When the play releases them from its grasp, learners experience the joy from solving problems. Dewey (1916) considered play to be purposeful activity that sponsors a child's growth. Teachers could capitalize on this for growth in learning. Learning to bring mathematical play into the classroom requires intention, an inviting attitude, knowledge of the types of problems that invoke play, and knowledge of how to connect playful problems to mathematical concepts and curriculum. Such engaging experiences with mathematics could sponsor joyful engagement in mathematics and an intrinsic desire to learn more.

Keywords: mathematical play; flow in mathematics; learning; joy

Le jeu et les mathématiques

Résumé :
Dans cet article, je propose que la manque d'enjouement et de joie dans nos salles de classes est causé par notre système d'éducation Nord-Américain, qui met l'accent sur les réussites scolaires. J'affirme que ce système n'est pas bénéfique pour pas la majorité d'apprenants, ni le domaine de mathématiques non plus. Beaucoup d'élèves sont affectés négativement—tant sur le plan émotionnel que scolaire—en mettant l'accent sur les résultats. Plutôt que de la pédagogie axée sur les résultats, une concentration sur l'apprentissage à aimer les mathématiques peut faire évoluer le discours. Une approche plus bienveillant axée sur les processus par le jeu mathématique peut susciter un enseignement et un apprentissage agréables. Le jeu (Gadamer, 1960/1989; Huizinga, 1944/1949) a le potentiel de captiver les apprenants qui cherchent des réponses à des problèmes de mathématiques ludiques mais difficiles. Semblable à l'état de flux, en jouant, les apprenants ont la chance de s'exercer et de développer leurs compétences existantes de manière à suspendre les notions de temps. Lorsque le jeu les relâches, les apprenants éprouvent la joie de résoudre des problèmes. Dewey (1916) considérait le jeu étant une activité résolue qui favorise la croissance de l'enfant; les enseignants pourraient ensuite en tirer parti pour améliorer leur apprentissage. Pour apprendre à intégrer le jeu mathématique en classe exige l'intention déterminée, une attitude invitante, une connaissance des types de problèmes qui invoquent le jeu ainsi qu'une bonne manière de relier ces problèmes ludiques à des concepts et à un programme d'études. De telles expériences captivantes avec les mathématiques pourraient susciter un engagement joyeux en mathématiques et un désir intrinsèque d'apprendre davantage.

Mots clés : jeu mathématique; l'état de flux en mathématiques; apprentissage; joie

With intention to play and an inviting attitude, a Grade 4 teacher introduced the following Jedi Knight problem:

> *After a battle, a Jedi Knight from the Old Republic, a Wookie, and two droids (C3PO and R2D2) are left stranded on the bridge of a damaged Imperial cruiser. They have 17 minutes to get off the bridge before it is engulfed in flame. The corridor leading from the bridge is in total darkness and so structurally weak that it will only hold two at a time. Using the Jedi's light saber as the light source, can all be rescued? The Jedi Knight can travel the corridor in one minute. The Wookie in two minutes. R2D2 in five minutes. C3PO in 10 minutes. Hint: Under usual conditions, no Jedi Knight would give his light saber to a Wookie or a droid, but . . .* (Galileo Education Network, 2019, "Extensions")

Students spent two minutes reading the problem silently to themselves, after which a few minutes were allotted for questions about the problem. Students then found partners and began trying to solve the problem. The classroom was noisy with banter. Each pair attempted many different combinations. The following is one such attempt:

The Jedi takes C3PO	→	*ten minutes*
The Jedi returns	→	*one minute*
The Jedi takes R2D2	→	*five minutes*
The Jedi returns	→	*one minute*
The Jedi takes Wookie	→	*two minutes*
Total:		*19 minutes*

Several students exclaimed, "It is impossible. It cannot be done." The teacher smiled slyly, "Oh yes, it can." For 40 minutes, the students worked noisily and diligently. The pairs volleyed back and forth, grouping different pairs of Star Wars characters. The bell for recess sounded. The students bellowed, "Nooooo. . . We are not done yet." The teacher said, "We'll try again tomorrow." After recess, a group of four students bounded towards the teacher, "We have it, we have it." They promptly told her their solution. The students had spent their recess solving the Jedi Knight problem.

For many, the above anecdote of gleeful enjoyment of mathematics in a classroom may seem unusual. How often are students so engrossed in a mathematics task that they do not want to stop and go to recess? How often do students willingly engage in mathematics problems during breaks? Why is there not more play and joy in mathematics classrooms?

In this paper, I argue that a lack of play and joy in classrooms could be due to our North American standardized education system, which emphasizes achievement outcomes. I argue that this system does not benefit the majority of students, nor the field of mathematics. While a few students experience success, many do not, and some are traumatized by mathematics. Consistent with the loving kindness topic of this special issue, I suggest mathematical play as a kinder way of teaching and learning mathematics. Perhaps, as Su (2017) suggests, play could be a way of flourishing in mathematics and beyond. In the next section, I first draw upon Huizinga (1944/1949) to highlight how play was lost through industrialization. Next, I draw upon Jardine, Clifford and Friesen (2008), as well as Davis, Sumara and Luce-Kapler (2006; 2015), to discuss the effects of industrialization and resulting standardized education.

Industrialization and a Loss of Play

Play existed before culture itself. Dutch cultural historian Johan Huizinga (1944/1949) noted that play was intertwined with ancient wisdom and is the main foundation of civilization. As an example of play in past culture, Brown (2008) pointed to a 15th century painting depicting a typical scene in a European courtyard, with all ages of people participating in 124 different kinds of play. Play was an integral part of society up until the 19th century and the advent of the Industrial Revolution. Huizinga (1944/1949) notes that

> the 19th century seem[ed] to leave little room for play. . . . [With] the Industrial Revolution and its conquests in the field of technology, work and production became the ideal, and then the idol, of the age. All Europe donned the boiler-suit. Henceforth the dominants of civilization were to be social consciousness, educational aspirations, and scientific judgement. (p. 192)

In the latter part of industrialization, and particularly during the Americanization of industrialization, Frederick Winslow Taylor's *Principle of Scientific Management* was popular. Assembly-line production, following the techniques developed by this engineer, promised to reduce or eliminate "wasted time . . . wasted energy, wasted materials and, in the end, wasted money and decreased productivity and profit" (Jardine, Clifford, & Friesen, 2008, p. xvi). In this context, workers' desires and emotions were seen as detrimental to productivity.

Loss of Play in Education

With a societal need to educate the masses, an industrialized model of education emerged that left little room for play. Around the same time that Taylor was developing his *Principles of Scientific Management*, psychologist Edward Thorndike was cultivating an analogous view of the nature of knowledge and knowledge acquisition (Jardine et al., 2008). Positing knowledge as a thing built up out of isolated, individual "experiences", Thorndike's pioneering theory of learning took hold as it aligned with the sensibilities of industrialization. Knowledge thus came to be understood as something to be assembled out of distinct bits and pieces, in a process analogous to the industrial production of an object out of its component parts. As Thorndike's work was later taken up by his successors, among them B. F. Skinner, behaviourism and the efficiency movement became intertwined (Jardine et al., 2008). One result was that the North American curriculum structure developed with an industrialized and behaviouristic view of education and knowledge.

In the early 1900s, the U.S. National Education Association applied Taylor's management model to the school system (Kliebard, 2004). Workers ("teachers") were handed simple instructions for repeated tasks, set to specific time limits and subject to "quality" control measures. Each teacher worked on an isolatable and definable piece of the overall task, so that if there were a problem with the end results, the specific site of the problem could presumably be found and fixed. For example, if a child had trouble with certain grammatical forms, the specific grade in which this was taught could be located and assessed for its effectiveness in the production of knowledge of this grammatical form (Jardine et al., 2008).

Within this framework, knowledge was broken into incremental developmental skills; teaching became the means by which these skills were dispensed, and learning was understood as the acquisition of these skills. With such standardized education, *knowing* is taken as an object; *learning* is taking things in; *intelligence* is measurable; *teaching* is conveying information; and the *learner*, characterized as deficient, is understood in terms of norms (Davis, Sumara, & Luce-Kapler, 2015).

Standardized Education's Stronghold

While behaviourisms rose to importance among researchers in the mid-1900s, they were inadequate for making sense of the complexity of human learning (Davis et al., 2015). Other learning theories such as inquiry and constructivism (e.g., Dewey, 1916; Piaget, 1954) emerged, which emphasized personal engagement "focused on understanding and rich inquiry" (Davis et al., 2015, p. 62). Davis et al. (2015) referred to this moment in education as *authentic education* and declared that with authentic education knowing is dynamic, accommodating, assimilating; learning is sense making, becoming aware, practicing; teaching is reorienting perceptions, facilitating, challenging; and the learner is developing.

Despite research-based evidence from the authentic education moment that provided insights into the complexity of learning, standardized education remained steadfast in the school system (Davis et al., 2015). Standardized education garnered more strength when the Russians launched the first artificial satellite, Sputnik, into orbit around the earth, thus allowing the Russians to gain an edge in the race in space against the Americans. Consistent with philosopher Bertrand Russell's (1918/1959) observation that mathematics is viewed as a means for "victory over foreign nations, whether in war or commerce" (p. 59), the space race prompted heated debates about how mathematics should be taught.

These heated debates were dubbed the Math Wars[1] and pitted traditionalists against reformers. Traditionalists promoted direct instruction, standardized methods and memorization. Reformers promoted inquiry—"problem solving . . . process[es] that actively engage[d] students in making conjectures, investigating and exploring ideas, discussing and questioning their own thinking and the thinking of others, validating results, and making convincing arguments" (as cited in Research Advisory Committee of the National Council of Teachers of Mathematics, 1988, p. 340). Typically, the rote (traditional) versus problem-solving (reform) Math Wars argument focuses on how students can achieve more in mathematics—particularly on large high-stakes international tests, such as the Programme for International Student Assessment (or PISA; see Organisation for Economic Cooperation and Development, 2018) and Trends in International Mathematics and Science Study (or TIMMS; see International Association for the Evaluation of Educational Achievement, 2019).

[1] See Schoenfeld (2004) for a more detailed review of the Math Wars.

In Canada's similarly post-industrialized world, the quest for efficiency and effectiveness in education has been and continues to be associated with ideas about accountability and standardized testing (Friesen, 2005). Thus, learning continues to be viewed in a standardized model of education, as acquisition, and mathematics is reduced to memorization of facts and procedures.

The Math Wars and the debates about how mathematics should be taught are still going strong—and the traditionalists may be winning (see Flanagan, 2018; Staples, 2014; Tran-Davies, 2018). Due to political pressure, Alberta and Manitoba have modified their curricula to include memorizing math facts (Alberta Education, 2016; Manitoba Education, 2013; Mertz, 2014). Further, Jason Kenney, Alberta's premier, aims to improve young Albertans' math proficiency by repealing the ministerial order that supported discovery, inquiry and constructivist learning in schools (Hare, 2019). Ontario's Ministry of Education has also published a new guide that focuses on the fundamentals (e.g., memorization) of mathematics (Ontario Ministry of Education, 2019).

The Impact of Standardized Education on Learners

The emphasis on outcomes neglects the emotional aspects of learning, in particular, that cognition and emotion are inextricably linked (Colombetti, 2013; Maturana & Varela, 1987; Varela, Thompson, & Rosch, 1991). Towers, Takeuchi and Martin (2018) found that "student's relationships with mathematics are fragile, volatile, subject to contextual influences, and constantly in process" (p. 161). They noted that a single moment of shame in early years may "propel a student to having a negative relationship with mathematics" (p. 161). More often than not, as Towers et al. (2018) found, students have negative relationships with mathematics.

When Takeuchi, Towers and Martin (2016) analyzed autobiographies of post-secondary students, they found that those who perceived mathematics as a set of procedures and facts tended not to pursue post-secondary mathematics courses. Those who perceived mathematics as beautiful, challenging and elegant pursued degrees in science, technology, engineering and mathematics (STEM)—areas that require mathematics. Towers et al.'s (2018) and Takeuchi et al.'s (2016) findings suggest that a pedagogical focus on students learning to enjoy doing mathematics might lead more students to pursue mathematics further. Noting that negative attitudes towards mathematics have been tied to lower achievement (Lipnevich, Preckel, & Krumm, 2016; Ma, 1999), I believe that that traditional teaching practices and curriculum are negatively affecting and propelling many students away from mathematics.

Changing the Conversation to a Kinder Curriculum

It is time to change the conversation from an outcome-focused curriculum to one based on flourishing, which I feel would be more kind and loving. Su (2017) has described five human desires that, when cultivated, may contribute to flourishing in mathematics: play, beauty, truth, justice and love. In this paper, I focus on play, as I consider it the foundational basis for flourishing in mathematics.

In the opening anecdote, the students' mathematical play led to positive experiences with challenging and elegant mathematics. Perhaps the positive experience also led them to dispositions that will enable them to embrace challenges, take risks, persevere, and thus deepen their conceptual understanding of mathematics. Perhaps, during those moments of play, the students were already flourishing. In the next sub-section, I explore definitions of play and what play means to learning.

Defining Play

Play is such a common word that everyone knows or thinks they know what play is. The etymology of the noun *play* is uncertain; it may come from the Middle Dutch verb *pleyen*, meaning to "dance, leap for joy, rejoice, be glad" (Play, 2018). According to Huizinga (1944/1949),

> play is a voluntary activity or occupation executed within certain fixed limits of time and place, according to rules freely accepted but absolutely binding, having its aim in itself and accompanied by a feeling of tension, joy and the consciousness that it is "different" from "ordinary life." Thus defined, the concept seem[s] capable of embracing everything we call "play" in animals, children and grown-ups: games of strength and skill, inventing games, guessing games, games of chance, and skill, exhibitions and performances of all kinds. We [venture] to call the category "play" one of the most fundamental in life. (p. 28)

Philosopher Hans-Georg Gadamer (1960/1989) picked up on Huizinga's notions of play in his own treatment of it. For Gadamer, play has its own life and exists beyond subjective experiences of it. As an example, the works of Shakespeare continue to exist long after his death. For centuries, his plays have enthralled audiences and will likely continue to do so indefinitely. According to Gadamer (1989), play has its own power. However, you cannot appreciate the play until you are drawn into it. In other words, you cannot be a spectator held aloof from the game; you have to let yourself be drawn into the play. Upon being drawn in, play holds the player in its grasp until the player is released.

The grip of play is productive. Brown (2008) described play as a state that allows the players to explore the possible. Su's (2017) more recent study described play as fun and as voluntary. For Su, play contains some structure but also freedom within the structure; play can lead to an investigation, with the possibility of surprise, but there is no great stake in the outcome.

When the player is in the throes of play and unconstrained from ordinary life, there is freedom to explore new ideas and possibilities. This exploration is not always joyful in itself, and there can be frustration when the new possibilities are not successful in the game. In the opening anecdote, the children made several incorrect attempts to solve the Jedi Knight problem. There were groans and expressions of "this is too hard", but the students were quick to try again. Failure was frustrating, but it did not deter the children from wanting to try to find a solution. Mistakes are part of the fun. It is this tension of exploration, of moving to and fro between ideas, that characterizes play. When the answer is found, or a game is won, the player experiences joy and elation upon finding the solution. These "aha" moments of joy fuel the players' motivation to play again.

Describing Play

In culture today, play is considered important for young children (e.g., Fisher, Hirsh-Pasek, Newcombe, & Golinkoff, 2013; Paley, 2007; Weisberg, Hirsh-Pasek, & Golinkoff, 2013; Zosh et al., 2017). However, play is also important for adults (Brown & Vaughan, 2009), and Csikszentmihalyi's (2000) notions of *flow* could provide insights into adult mathematical play. Csikszentmihalyi developed his notions of flow as he watched artists work for hours painstakingly focused on creating their works of art. The artists appeared more interested in the process than the finished product. Thus, the artists placed their finished works aside and started again. Csikszentmihalyi termed the process of giving your heart, mind, body and soul over to the work as being "in the flow". Flow is characterized by intense concentration on an achievable goal or task with deep, yet effortless immersion. To enter a flow state, one must have autonomy over the situation or activity. While in a flow state, one loses self-consciousness, experiences pleasure, becomes energized, and often loses a sense of time. In this way, flow recalls Gadamer's (1989) notion that play has its own power.

As with Gadamer's (1960/1989) notion of play, flow is entered into voluntarily, yet is completely absorbing. Both play and flow feel agential; in either case, the individual is held in their grasp until released. And yet, while experiencing flow, the individual has the freedom to try new ideas and explore possibilities. As with Su's (2017) notions of play, flow is fun and voluntary, containing some structure but with freedom within the structure. As with play, flow can also lead to an investigation with the possibility of surprise, and there is no great stake in the outcome. The experience of flow could be considered part of a life well-lived, with what Su (2017) would describe as flourishing.

Play and Mathematics

The mathematician Henri Poincaré wrote about an experience that could be described as an instance of flow, or of mathematical play:

> For fifteen days I strove to prove that there could not be any functions like those I have since called Fuchsian functions. I was then very ignorant; every day I seated myself at my table, stayed an hour or two, tried a great number of combinations and reached no results. One evening, contrary to my custom, I drank black coffee and could not sleep. Ideas rose in crowds; I felt them collide until pairs interlocked, so to speak, making a stable combination. By the next morning I had established the existence of a class of Fuchsian functions, those which come from the hypergeometric series; I had only to write out the results, which took but a few hours. (as cited in Stewart, 2006, pp. 55-56)

Poincaré's description of this experience is similar to those of flow: he was completely absorbed in the task, was concentrating intensely and lost track of time. Flow is analogous to Huizinga's (1949) notion of play, where individuals voluntarily abandon themselves in the work, follow a clear task with defined rules, get caught up in the task, and are not released until the task is done—or, as in mathematics, until the problem is solved. From Poincaré's experience, he described how "the answer to an enigmatic question is not found by reflection or logical reasoning. It comes quite literally as a sudden solution—a loosening of the tie by which the questioner holds you bound" (as cited in Stewart, 2006, p. 110). Poincaré found the solution to Fuchsian functions with a sudden insight. The riddle had held Poincaré in its grasp and did not release him until he had found its secret.

Play and Learning

Flow is associated with positive experiences and intrinsic motivation when learning mathematics. Similarly to play, when experiencing flow (Csikszentmihalyi, 2000), learners get a chance to practice and elaborate on their existing skills in manners that suspend notions of time. In a case study, Williams (2004) studied one Grade 8 student, Leon, who was learning to understand the area of a triangle. Leon identified the challenge and delved into the issue with manners consistent with flow. In interviews, Leon explained how he switched from being worried about the answer to focusing on the process. Williams attributed Leon's identification of the challenge, concept development and positive affect as indicative of flow experiences. The experience of flow changes the pedagogical task into a process of learning to understand, rather than just finishing the work required.

In a three-part longitudinal study of 220 seventh- and eighth-graders, Heine (1997) studied the relationship among mathematical achievement, self-reports of flow and mathematical ability. His findings suggest that flow is significantly better than extrinsic motivation for predicting achievement; that extrinsic motivation is associated negatively with achievement; and that achievement is dependent on ability and task enjoyment, but not on cognitive ease. In mathematics classrooms that afforded more complex opportunities, students experienced significantly more flow, and their achievement was half a grade higher than students in less complex mathematics classrooms (Heine, 1997).

In another study, Golnabi (2017) surveyed 113 undergraduate students in a developmental algebra class. When students with strong self-efficacy practiced problems, they reported positive flow-like feelings—they felt that the task presented "just the right amount of challenge" and they became "completely lost in thought" (Golnabi, 2017, p. 59). Encouraging flow experiences in classrooms is considered desirable for positive, intrinsically motivated learning experiences (Borovay, Shore, Caccese, Yang, & Hua, 2019; Golnabi, 2017; Heine, 1997; Williams, 2004).

Returning to Play

Play is vital to our well-being and happiness (Brown & Vaughan, 2009; Su, 2017), and it can be a means of seeking knowledge. Huizinga (1949), for one, reminds us that knowledge and play were once intrinsically bound. For example, the sacred riddle contests in ancient civilizations tested the wisdom of the sages. Here, play was voluntary, risky, serious and bound by fixed rules. Play also provides active, engaged, meaningful and socially interactive experiences that are known to be conducive to learning (Hirsh-Pasek et al., 2015; Weisberg et al., 2013).

Teaching Play

For American philosopher John Dewey (1916), play is a purposeful and spontaneous activity that sponsors a child's growth. Dewey claimed that for the child, play and work are one and the same. It is their occupation, one in which they can be thoroughly engaged and absorbed. Dewey (1916) explained: "Education has no more serious responsibility than making adequate provision for enjoyment of recreative leisure; not only for the sake of immediate health, but still more if possible for the sake of its lasting effect upon habits of mind" (Chapter 15, Section 3, para. 4).

Dewey (2016) thought that teachers could harness a child's innate willingness to play by orienting attentions. He cautioned, however, that too much teacher direction could suppress emerging play. Su (2017) cautions that teaching play is more challenging than lecturing because the teacher must be able to respond to anything that could happen in the classroom. But Su (2017) also noted that "it's more fun" ("Play," para. 8) and that students learn better: "Everyone can play. Everyone enjoys play. Everyone can have a meaningful experience in mathematical play. If play is essential for flourishing in mathematics, then the play in teaching and learning should be encouraged" (Su, 2017, "Play," para. 7).

Teachers can utilize learners' innate tendencies towards play to make mathematics learning fun and long lasting. Su (2017) argued that an intention to teach play in classrooms changes the purpose of teaching mathematics from the delivery of information to the engagement in mathematical play, which enables students to flourish in mathematics. He cautioned that "this way [of teaching] will be no less rigorous and no less demanding of our students. And yet it will draw more people into mathematics because they will see how mathematics connects to their deepest human desires" ("Why do mathematics?", para. 5).

For mathematical play to exist, there must be players and a structure or a playground to play with, and the pedagogical knowledge necessary to engage such play is what Davis et al. (2000) have called structuring occasions for play. As such, the classroom should be organized so that interactions allow for connections and insights that can propel learning forward. In this context, learning is considered a playful web of emergent possibilities: knowing is being, and teaching is occasioning, engaging and mindful participation. Bringing mathematical play into the classroom requires knowing the types of problems that invoke play, having a playful attitude, and knowing how to connect playable problems to mathematical concepts and curriculum (Francis-Poscente, 2009).

The types of problems selected by the teacher for structuring occasions for play are crucial. To be played, a mathematical problem needs to hold the player in its grasp. The problem itself needs to be able to draw the player in and hold the player firmly in its grasp, with secrets hidden from the player. With such a problem, the players will get caught up in the seriousness of the problem. To be able to hold a player in its grasp, the problem needs to have enough challenge to hold interest but not be so challenging that a child cannot get caught up in it.

Not all mathematical problems, however, present themselves in manners that can be played. Most mathematical problems found in schools are routine exercises designed for memorizing facts and algorithms (Schoenfeld, 1992). Gerofsky (1999) suggested that school word problems tend to focus on the "real world" in order to be realistic and, thus, most relevant for students learning "serious mathematics". But school word problems, Gerofsky (1999) noted, are so focused on mundane details that they are tedious and far removed from actual, lived experiences. Assigned as "bitter medicine" to test student competence, word problems "are ideally meant to be solved silently, individually, using pen and paper" (p. 40). Fuchs et al. (2008) considered school word problems to often be more of a linguistic exercise disguising simple mathematical exercises. As such, any playability of such problems is stymied.

Gerofsky (1999) also discussed the distinction between riddles, recreational puzzles and school word problems. The difference, she noted, lies in their intention: riddles and recreational puzzles are for entertainment and can potentially hook students who feel they are not strong in math and might otherwise be scared away.

The Jedi Knight problem in the opening anecdote is an example of a type of recreational puzzle problem. This problem is a rewrite of an old problem more commonly known as the River Crossing Problem, whose first written origins can be traced to the 8th century (Danesi, 2004). This problem has for centuries entertained by puzzling. The longevity of this problem (and many others) may be an indication of Gadamer's (1960/1989) notion that play has a power that extends beyond subjective experiences of the player. The problem continues long after its invention and remains in play regardless of who plays it. This longevity may also be an indication of the love of mathematics that these problems inspire, as it is with loving care that these puzzle problems are handed from one generation to another.

The children in the opening anecdote played with mathematics together. Working in pairs, the Jedi Knight problem held their attention in class and then through recess. The students were absorbed in the task. Held in the grasp of play, the students went back and forth, trying a large number of combinations to get everyone across the bridge before it collapsed in 17 minutes—without a correct result. But at some time during recess, a group of four was inspired to try a previously untried combination. In finding the solution, the students were released from the play. They were excited and could not wait to share their insights with the teacher.

Each combination that was tried by the students required several calculations and comparisons. Their problem solving illustrated the mathematics the students engaged in. In the problem, pairs travel at the slowest traveler's rate, therefore each selection of pairs requires a comparison of which speed is lowest. There are six possible combinations for just the first crossing. In the solution, there are three comparisons of inequality to be made. Then the total time requires adding five numbers together and comparing to 17 minutes—another comparison of inequality or equality. The students voluntarily tried multiple combinations for the entire class time. The students also appeared joyful as they worked on solving this problem. The anecdote provides insight into how engaging with mathematical play could sponsor joyful engagement in mathematics and an intrinsic desire to learn more mathematics.

Conclusion

In summary, play can lead to meaningful, positive and fun learning experiences. In my opinion, play has the potential to enthrall children in meaningful mathematical challenges. Play shifts the focus from outcomes to a focus on processes where children have freedom to explore different concepts and connect mathematical ideas. Through this focus on process, children develop a willingness to welcome mathematical challenges which will lead to stronger mathematical insights and conceptual development. With positive playful experiences with mathematics, students could learn to see mathematics as beautiful, challenging and elegant. I believe that play could lead to flourishing in mathematics and beyond.

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